9

$$-3y + 2\frac{d}{dx}y + \frac{d^2}{dx^2}y = xe^x + 1$$

$$y_c : -3y + 2\frac{d}{dx}y + \frac{d^2}{dx^2}y$$
$$= 0$$

$$r^2 + 2r - 3$$

$$r : [1], [-3]$$

$$y(x) = C_1 e^{-3x} + C_2 e^x$$

$$xe^x$$

$$1$$

$$e^x \to x^2 e^x$$

$$y_p = C_3 x e^x + C_4 + C_5 x^2 e^x$$

$$y_p' = C_3 x e^x + C_3 e^x + C_5 x^2 e^x + 2C_5 x e^x$$

$$y_p'' = C_3 x e^x + 2C_3 e^x + C_5 x^2 e^x + 4C_5 x e^x + 2C_5 e^x$$

$$-3C_3 x e^x - 3C_4 - 3C_5 x^2 e^x + 2C_3 x e^x + 2C_3 e^x + 2C_5 x^2 e^x$$
$$+ 4C_5 x e^x + C_3 x e^x + 2C_3 e^x + C_5 x^2 e^x + 4C_5 x e^x + 2C_5 e^x = xe^x + 1$$

$$4C_3 e^x - 3C_4 + 8C_5 x e^x + 2C_5 e^x = xe^x + 1$$

$$4C_3 + 2C_5 = 0$$

$$8C_5 = 1$$

$$-3C_4 = 1$$

$$C_4 = -\frac{1}{3}$$

$$C_5 = \frac{1}{8}$$

$$C_3 = -\frac{1}{16}$$

$$y_p = \frac{x^2 e^x}{8} - \frac{xe^x}{16} - \frac{1}{3}$$

$$y = C_1 e^{-3x} + C_2 e^x + \frac{x^2 e^x}{8} - \frac{xe^x}{16} - \frac{1}{3}$$

10

$$9y + \frac{d^2}{dx^2}y = 3\sin(3x) + 2\cos(3x)$$

$$y_c : 9y + \frac{d^2}{dx^2}y$$
$$= 0$$

$$r^2 + 9$$

$$r : [-3i], [3i]$$

$$y(x) = C_1 \sin(3x) + C_2 \cos(3x)$$

$$\cos(3x) \to x\cos(3x)$$

$$\sin(3x) \to x\sin(3x)$$

$$y_p = C_3 x\cos(3x) + C_4 x\sin(3x)$$

$$y_p' = -3C_3 x\sin(3x) + C_3\cos(3x) + 3C_4 x\cos(3x) + C_4\sin(3x)$$

$$y_p'' = -9C_3 x\cos(3x) - 6C_3\sin(3x) - 9C_4 x\sin(3x) + 6C_4\cos(3x)$$

$$9C_3 x\cos(3x) + 9C_4 x\sin(3x) + -9C_3 x\cos(3x) - 6C_3\sin(3x) - 9C_4 x\sin(3x) + 6C_4\cos(3x)$$
$$= 3\sin(3x) + 2\cos(3x)$$

$$-6C_3\sin(3x) + 6C_4\cos(3x) = 3\sin(3x) + 2\cos(3x)$$

$$-6C_3 = 3$$

$$6C_4 = 2$$

$$C_4 = \frac{1}{3}$$

$$C_3 = -\frac{1}{2}$$

$$y_p = \frac{x}{3}\sin(3x) - \frac{x}{2}\cos(3x)$$

$$y = C_1\sin(3x) + C_2\cos(3x) + \frac{x}{3}\sin(3x) - \frac{x}{2}\cos(3x)$$

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$$5y + 2\frac{d}{dx}y + \frac{d^2}{dx^2}y = e^x \sin(x)$$

$$y_c : 5y + 2\frac{d}{dx}y + \frac{d^2}{dx^2}y$$
$$= 0$$

$$r^2 + 2r + 5$$

$$r : [-1 - 2i], [-1 + 2i]$$

$$y(x) = (C_1 \sin(2x) + C_2 \cos(2x)) e^{-x}$$

$$e^x \sin(x)$$

$$e^x \cos(x)$$

$$y_p = C_3 e^x \sin(x) + C_4 e^x \cos(x)$$

$$y_p' = C_3 e^x \sin(x) + C_3 e^x \cos(x) - C_4 e^x \sin(x) + C_4 e^x \cos(x)$$

$$y_p'' = 2C_3 e^x \cos(x) - 2C_4 e^x \sin(x)$$

$$5C_3 e^x \sin(x) + 5C_4 e^x \cos(x) + 2C_3 e^x \cos(x) - 2C_4 e^x \sin(x)$$
$$+ 2C_3 e^x \sin(x) + 2C_3 e^x \cos(x) - 2C_4 e^x \sin(x) + 2C_4 e^x \cos(x) = e^x \sin(x)$$

$$7C_3 e^x \sin(x) + 4C_3 e^x \cos(x) - 4C_4 e^x \sin(x) + 7C_4 e^x \cos(x) = e^x \sin(x)$$

$$4C_3 + 7C_4 = 0$$

$$7C_3 - 4C_4 = 1$$

$$C_4 = -\frac{4}{65}$$

$$C_3 = \frac{7}{65}$$

$$y_p = \frac{7e^x}{65} \sin(x) - \frac{4e^x}{65} \cos(x)$$

$$y = (C_1 \sin(2x) + C_2 \cos(2x)) e^{-x} + \frac{7e^x}{65} \sin(x) - \frac{4e^x}{65} \cos(x)$$